

Worksheet 2

ALT II

$$1. \quad \begin{aligned} x - y &= -1 & x &= y - 1 \\ x + y &= 3 \end{aligned}$$

$$\begin{aligned} (y - 1) + y &= 3 \\ 2y - 1 &= 3 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

$$\begin{aligned} x - 2 &= -1 \\ x &= 1 \\ x + 2 &= 3 \\ x &= 1 \end{aligned}$$

$$\boxed{(1, 2)}$$

$$2. \quad \begin{aligned} x - y &= -1 & x &= y - 1 \\ 5y - 5x &= 10 \end{aligned}$$

$$\begin{aligned} 5y - 5(y - 1) &= 10 \\ 5y - 5y + 5 &= 10 \\ 5 &= 10 \end{aligned}$$

$$\boxed{\text{No Solution}}$$

$$3. \quad \begin{aligned} y &= x^2 - 4 \\ y - 2x &= -5 \end{aligned}$$

$$\begin{aligned} (x^2 - 4) - 2x &= -5 \\ x^2 - 2x - 4 &= -5 \\ x^2 - 2x + 1 &= 0 \\ (x - 1)(x - 1) &= 0 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} y &= (1)^2 - 4 \\ y &= 1 - 4 \\ y &= -3 \\ y - 2(1) &= -5 \\ y - 2 &= -5 \\ y &= -3 \end{aligned}$$

$$\boxed{(1, -3)}$$

$$4. \begin{cases} y = x^2 - 4 \\ y - 2x = -6 \end{cases}$$

$$(x^2 - 4) - 2x = -6$$

$$x^2 - 2x - 4 = -6$$

$$x^2 - 2x + 2 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-4}}{2}$$

No Solution

$$5. x^2 - 3y^2 = -11$$

$$x - y = -1 \quad x = y - 1$$

$$(y-1)^2 - 3y^2 = -11$$

$$y^2 - 2y + 1 - 3y^2 = -11$$

$$-2y^2 - 2y + 1 = -11$$

$$-2y^2 - 2y + 12 = 0$$

$$-2(y^2 + y - 6) = 0$$

$$-2(y+3)(y-2) = 0$$

$$y = -3, 2$$

$$\begin{aligned} (-4, -3) \\ (1, 2) \end{aligned}$$

$$x^2 - 3(-3)^2 = -11$$

$$x^2 - 3(9) = -11$$

$$x^2 - 27 = -11$$

$$x^2 = 16$$

$$x = 4 \text{ or } x = -4$$

$$x - (-3) = -1$$

$$x + 3 = -1$$

$$x = -4$$

$$x^2 - 3(2)^2 = -11$$

$$x^2 - 3(4) = -11$$

$$x^2 - 12 = -11$$

$$x^2 = 1$$

$$x = 1 \text{ or } x = -1$$

$$x - (2) = -1$$

$$x - 2 = -1$$

$$x = 1$$

$$6. x^2 + y^2 = 9$$

$$y = x + 1$$

$$x^2 + (x+1)^2 = 9$$

$$x^2 + x^2 + 2x + 1 = 9$$

$$2x^2 + 2x + 1 = 9$$

$$2x^2 + 2x - 8 = 0$$

$$2(x^2 + x - 4) = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{17}}{2}$$

$$x = \frac{-1 + \sqrt{17}}{2} \quad \text{or} \quad x = \frac{-1 - \sqrt{17}}{2}$$

$$x = -\frac{1}{2} + \frac{1}{2}\sqrt{17} \quad \text{or} \quad x = -\frac{1}{2} - \frac{1}{2}\sqrt{17}$$

$$\left(-\frac{1}{2} + \frac{1}{2}\sqrt{17}, \frac{1}{2} + \frac{1}{2}\sqrt{17}\right)$$

$$(1.56, 2.56)$$

and

$$\left(-\frac{1}{2} - \frac{1}{2}\sqrt{17}, \frac{1}{2} - \frac{1}{2}\sqrt{17}\right)$$

$$(-2.56, -1.56)$$

$$y = \left(\frac{-1 + \sqrt{17}}{2}\right) + 1$$

$$y = \frac{1}{2} + \frac{1}{2}\sqrt{17}$$

$$\left(-\frac{1}{2} + \frac{1}{2}\sqrt{17}\right)^2 + \left(\frac{1}{2} + \frac{1}{2}\sqrt{17}\right)^2 \stackrel{?}{=} 9$$

true

$$y = \left(-\frac{1}{2} - \frac{1}{2}\sqrt{17}\right) + 1$$

$$y = \frac{1}{2} - \frac{1}{2}\sqrt{17}$$

$$\left(-\frac{1}{2} - \frac{1}{2}\sqrt{17}\right)^2 + \left(\frac{1}{2} - \frac{1}{2}\sqrt{17}\right)^2 \stackrel{?}{=} 9$$

true

7. Hassan is an artist who specializes in geometric designs. He is trying to get ready for a street fair next month. Hassan paints both watercolors and pastels. Each type of picture takes him about the same amount of time to paint. He figures he has time to make a total of at most 16 pictures.

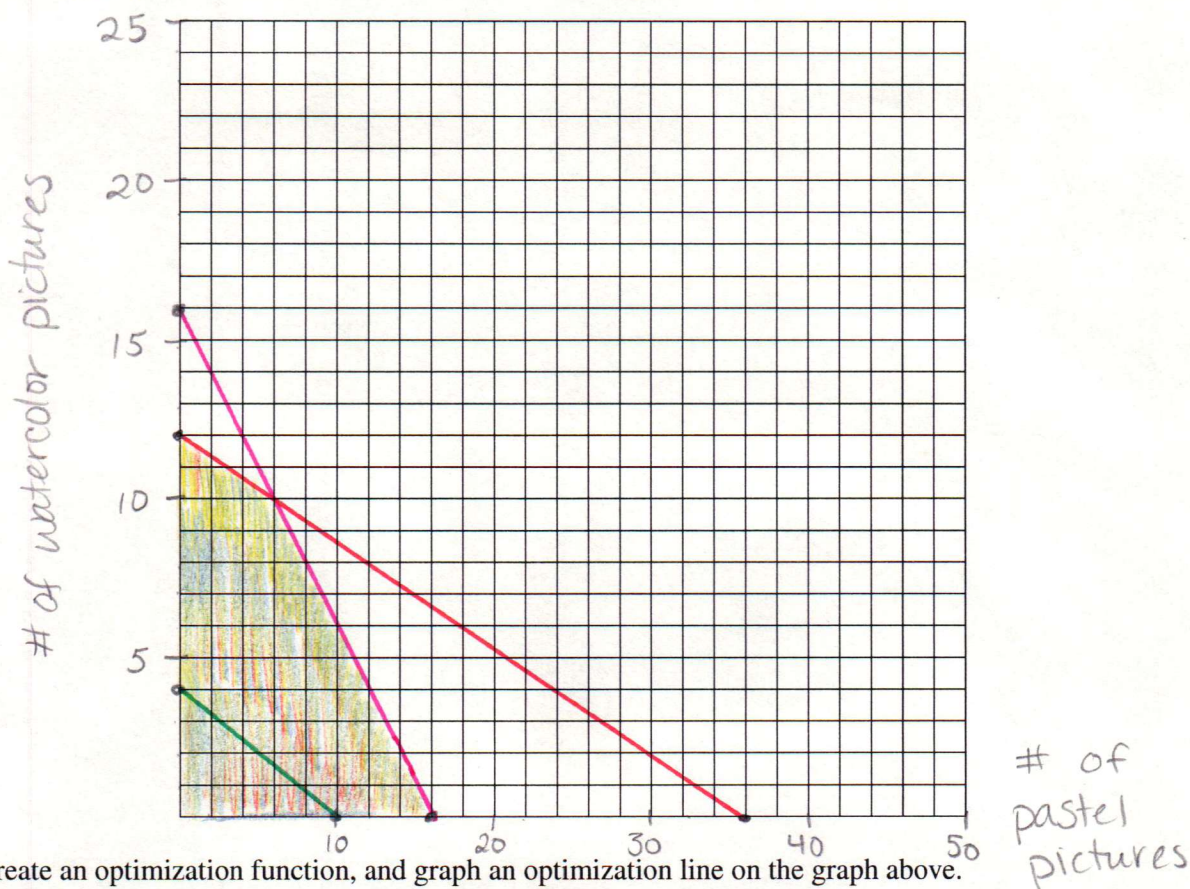
The materials for each pastel will cost him \$5 and the materials for each watercolor will cost him \$15. He has \$180 to spend on materials. Hassan makes a profit of \$40 on each pastel and a profit of \$100 on each watercolor. How many paintings of each type should he create to maximize his profit? What is that profit?

- (a) Carefully define two relevant variables for this situation. (b) Carefully write a complete set of constraints.

$x = \#$ of pastel pictures
 $y = \#$ of watercolor p

* $x + y \leq 16$ $x: 16$ $y: 16$
 * $5x + 15y \leq 180$ $x: 36$ $y: 12$
 $x \geq 0, y \geq 0$

- (c) Carefully graph the constraints and shade the feasible region.



- (d) Create an optimization function, and graph an optimization line on the graph above.

$40x + 100y = \text{profit}$ * $40x + 100y = 400$
 $x: 10$ $y: 4$

- (e) Determine the optimal point and answer the questions.

Hassan should make 6 pastel pictures and 10 watercolor pictures for a maximum profit of \$1240.

$$x + y = 16$$

$$x = -y + 16$$

$$(6, 10)$$

or

$$(0, 12)$$

$$5x + 15y = 180$$

$$5(-y + 16) + 15y = 180$$

$$-5y + 80 + 15y = 180$$

$$10y = 100$$

$$y = 10$$

$$x + 10 = 16$$

$$x = 6$$

$$40x + 100y = \text{profit}$$

$$40(6) + 100(10) = \text{profit}$$

$$\$1240$$

$$40(0) + 100(12) = \text{profit}$$

$$\$1200$$